J. Pure Appl. & Ind. Phys. Vol.3 (2), 110-114 (2013)

An Evaluation of Condensation temperature T_c and Condensate fraction $\left(\frac{N_o}{N}\right)$ as a Function of $\left(\frac{T}{T_c}\right)$ for Bose-Einstein Condensation of Trapped Atomic Gas Using Non Extensive Statistical Mechanics

V. K. VERMA

Department of Physics,
Gaya College Gaya, Bihar, INDIA.

(Received on: February 11, 2013)

ABSTRACT

Using theoretical formalism developed by Wang *et al*.[5] we have theoretically evaluated the transition temperature T_c of the Bose-Einstein condensation. Our evaluated results of T_c for three Bose condensates are lower than the observed T_c . We have also evaluated condensate fracton $\left(\frac{N_o}{N}\right)$ as a function of $\left(\frac{T}{T_c}\right)$for three values of non extensive parameter q. $\left(\frac{N_o}{N}\right)$decrease with $\left(\frac{T}{T_c}\right)$ for all the three values of q as q=1.1,1.0and 0.8. Our theoretically evaluated results are in good agreements with that of the other theoretical workers.

Keywords: Bose-Einstein Condensation, interacting system, quantum particle systems.

1. INTRODUCTION

The creation of Bose-Einstein Condensation (BEC) in dilute atomic gases[1-3] of ^{87}Rb, ^{7}Li, ^{23}Na and others has generated a great deal of interest in the statistical investigation of interacting (imperfect) quantum particle systems. It is realised that the conventional Bose-Einstein Statistics (BES) fails to yield the observed transitation temperature T_c. A good example is[4] He for which the observed transition temperature T_c=2.17K and theoretical one T_c=3.10K. For the dilute atomic gases trapped in harmonic potential, there are also significant differences between the observed and theoretical T_c. In addition the non-interacting gas picture gives an atom-velocity distribution of the condensate which is not consistent with that observed in the atomic

vapour[1,3]. These failures are obviously due to neglect of the interacton due to particle in BES. One of the treatments taking into account this interaction is proposed by Huang[4]. Huang has proposed the following approxiamation at low temperature

a) The particle interaction takes place through binary collision which is known as short term interaction.
b) The effective interaction is weak.
c) The particle sees only average effect of the interaction which is mean field theory.
d) Only first order perturbation is considered.

On the basis of the above approximation Huang has obtained the interaction energy $\delta U = \frac{4\pi\hbar^2 a}{m}$

Where a is the scattering length of s-wave of the atomic collision and m is the particle mass. In general 'a' is positive for repulsive interaction and negative for attractive interaction. It was observed in the above experiment[1-3] of BEC the values of 'a' obtained for ^{87}Rb, a=200a_0 (repulsive), for ^{23}Na a=(92±25) a_0 and for ^{7}Li gas, a=(-27±0.8)a_0,a_0 is the Bohr radius. It shows that this theory has problem with ^{7}Li gas because a<o. This leads to imaginary physical quantities of the gas. This is unphysical, this tells that BEC of ^{7}Li dilute gas is impossible.

Using the theoretical formalism develped by Wang etal[5], we have evaluated the generalised transition temperature T_{cq} and condensate fraction $\frac{N_o}{N}$ as a function of $\frac{T}{T_c}$ for different values of q. Wang etal[5] have studied the interacting quantam gas within non extensive Statistical mechanics (NSM)[6-8] which is considered as a possible theory for interacting system. We have found that the theoretically evaluated values of T_c is lower than the observed T_c for the above three gases. Our theoretical analysis also indicates that there should be attractive interaction in the three condensates.

2. MATHEMATICAL FORMULAE USED IN THE EVALUATION

One uses non extensive statistical mechanics (NSM) as a possible theory for interacting system, Within NSM, a bosm distribution[8] is given by

$$\langle n_q \rangle = \frac{1}{\left[1+(q-1)\frac{\mathcal{E}-\mu}{K_\beta T}\right]^{\frac{1}{q-1}}-1} \quad (1)$$

Where n_q is the average occupation number at a state with energy $\mathcal{E}$ and chemical potential μ. The parameter q is positive real number. In this case q≠1 the internal energy of the system varies as q changes[9-10]. The interactions are repulsive (or attractive) for q>1 (or q<1). This formalism can be applied to the dilute gases of ^{87}Rb, ^{7}Li and ^{23}Na atoms trapped in harmonic potentials[1-3]. For imperfect boson gas trapped in a three dimensional harmonic potential

$$V(X,Y,Z) = \frac{1}{2}K(X^2+Y^2+Z^2)$$
$$= \frac{1}{2}KR^2 \quad (2)$$

The total number of particles of mass m is given by

$$N = \frac{2(4\pi)^2}{h^3}\left(\frac{m}{K}\right)^{\frac{3}{2}}(K_\beta T)^3$$
$$\int_0^{x_q}\int_0^{y_q} \frac{x^{\frac{1}{2}}dx\, y^{\frac{1}{2}}dy}{[1+(1-q)(x+y-\nu)]^{\frac{1}{q-1}}-1} \quad (3)$$

Where $= \frac{p^2}{2mK_\beta T}$, $y = \frac{KR^2}{2K_\beta T}$, $\nu = \frac{\mu}{K_\beta T}$

For q<1, $\left(x_q + y_q\right) = \frac{q}{1-q} + \nu$

For q>1, $x_q = \infty$ and y_q depends on the living space of the particles. If one let the quantum mechanical living space tend to infinity[11], then $y_q \rightarrow \infty$. For large x and y, q must be limited in order that the integration (3) converges.

Now, for large x and y

$$\frac{x^{\frac{1}{2}}y^{\frac{1}{2}}}{(x+y)^{\frac{1}{q-1}}} = \frac{x^{\frac{1}{2}}}{(x+y)^{\frac{1}{2(q-1)}}} \cdot \frac{y^{\frac{1}{2}}}{(x+y)^{\frac{1}{2(q-1)}}}$$

$$\leq \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2(q-1)}}} \cdot \frac{y^{\frac{1}{2}}}{y^{\frac{1}{2(q-1)}}}$$

Now, this leads to $q < \frac{4}{3}$ for the convergence of the integral and $q < \frac{5}{3}$ for free particle model.

Then equation (3) can be written as $N = N_o + QI_qN(O)$ (4)

Where N is the total number of trapped particles. N_o is the occupation number of the ground state.

$$N_o = \frac{1}{\left[1+(q-1)\left(\frac{3}{2}\hbar\omega - \nu\right)\right]^{\frac{1}{q-1}} - 1}$$

And $\varepsilon = \frac{3}{2}\hbar\omega$, $Q = \left[\frac{K_\beta T}{\hbar\omega}\right]^3$ (5)

And

$$I_qN(o) = \frac{4}{\pi}\int_0^{x_q}\int_0^{y_q} \frac{x^{\frac{1}{2}}dx\, y^{\frac{1}{2}}dy}{[1+(q-1)(x+y-\nu)]^{\frac{1}{q-1}}-1} \quad (6)$$

The Critical temperature T_{cq} of Bose-Einstein condensation is defined by

$$I_qN(o) = \frac{N}{Q} \quad (7)$$

$$K_\beta T_{cq} = \hbar\omega\left(\frac{N}{I_qN(o)}\right)^{\frac{1}{3}} \quad (8)$$

Here ω is the average trapping frequency, $\hbar$ the Planck Constant and K_β is the Boltzmann Constant.

Now, $I_qN(o) = \frac{4}{3}\int_0^{x_q}\int_0^{y_q} \frac{x^{\frac{1}{2}}dx\, y^{\frac{1}{2}}dy}{[1+(q-1)(x+y)]^{\frac{1}{q-1}}-1}$ (9)

For q=1, I N(o)= 1.202, $\frac{T_{cq}}{T_c} = \left[\frac{1.202}{I_qN(o)}\right]^3$

This ratio is zero for q=1.333.

Now for $T\left(\frac{T_{cq}}{\mu} = 0\right)$

One can write $\frac{N_o}{N} = 1 - \frac{I_qN(o)}{\propto}$ (10)

Where α is given by $\propto = \frac{N}{Q} = N\left(\frac{\hbar\omega}{K_\beta T}\right)^3$ (11)

With these two equations one can write

$$\frac{N_o}{N} = 1 - \left(\frac{T}{T_{cq}}\right)^3 \quad (12)$$

This gives the percentage of condensed particle when $T<T_{cq}$. In this case when q=1, equ^n(7) gives the convensional result of BEC.

$$I_q(o) = N\left(\frac{\hbar\omega}{K_BT_c}\right)^3 = 1.202$$

Or, $T_c = \frac{\hbar\omega}{K_B}\left(\frac{N}{1.202}\right)^{\frac{1}{3}}$ (13)

Then from equ^n (7) and (13) one can find out relation between generalised critical temprature T_{cq} and convensional T_c

$$T_{cq} = T_c\left(\frac{1.202}{I_q(o)}\right)^{\frac{1}{3}} \quad (14)$$

Then equation (12) can be written as

$$\frac{N_o}{N} = 1 - \frac{I_qN_{(o)}}{1.202}\left(\frac{T}{T_c}\right)^3 \quad (15)$$

DISCUSSION OF RESULTS

In this paper, using theoretical formalism of Wang *et al.*[5], we have theoretically evaluated the transition temperature T_c from eq^n (13) for three Bose gases namely ^{87}Rb, ^{23}Na and ^{7}Li. These results were compared with the observed transition temperature of the Bose gas systems[1-3]. Wang etal[5] formalism is based on the application of non-extensive statistical mechanics[6-8]. In the Bose-Einstein

Condensation interacting atoms or molecules the theoretical understanding of transition temperature is not always obvious due to the interaction of zero point energy which can not be exactly taken into account. The s-wave collision model fails to account for the condensation temperature. Hence using equ^n(13) we have evaluated transition temperature T_c of all the three Bose gases which is shown in table T_1 along with the observed T_c. We find that the theoretical results are always less than the observed T_c in all the three cases. We have also evaluated the values of non-extensive parameter q. We find q=0.1 for ^{87}Rb atomic gas, q=0.62 for ^{23}Na and q=0.95 for ^{7}Li. This shows that the effective interaction are essentially attractive in all the three considered atoms. We have also evaluated condensate fraction $\left(\frac{N_o}{N}\right)$ as a function of $\left(\frac{T}{T_c}\right)$ for three values of q. The results are shown in table T_2. Our theoretically evaluated results for $\left(\frac{N_o}{N}\right)$ decreases as a function of $\frac{T}{T_c}$. The decrease is more pronounced for q=1.1, These results are in the good agreement with those of the other theoretical workers[12-15].

Table T_1

Theoretical T_c calculated from equ^n (13) N_c (no. of particles observe at transition) T_{cq} (observed condensation temperature) $I_qN(o)$, the value of q

	T_c	N_c	T_{cq}	$I_qN(o)$	q
^{87}Rb gas	74 nk	$2X10^4$	170 nk	0.99	q=0.1
^{23}Na gas	1350 nk	$15X10^6$	2000nk	0.37	q=0.62
^{7}Li gas	386 nk	$2X10^5$	400 nk	1.08	q=0.95

Table T_2

An evaluated result of $\left(\frac{N_o}{N}\right)$ as a function of $\left(\frac{T}{T_c}\right)$ for three different values of q from equ^n (15).

$\frac{T}{T_c}$	$\leftarrow\left(\frac{N_o}{N}\right)\rightarrow$		
	q=1.1	q=1.0	q=0.8
0.1	1.00	1.00	1.00
0.2	0.953	0.967	0.982
0.3	0.926	0.938	0.942
0.4	0.867	0.886	0.906
0.5	0.735	0.805	0.835
0.6	0.586	0.762	0.785
0.7	0.432	0.635	0.695
0.8	0.278	0.581	0.602
0.9	0.186	0.455	0.586
1.0	0.095	0.316	0.432
1.1	0.032	0.215	0.252
1.2	0.006	0.108	0.147
1.5	0.008	0.059	0.087

ACKNOWLEDGEMENT

I express my sincere thanks to Prof. L.K. Mishra, Deptt. of Physics, Magadh University Bodh-Gaya, for very useful suggestions and discussions in this paper.

REFERENCES

1. M.H. Anderson, J.R Ensher, M.R. Matthews, C.E. Wieman and L.A. Cornell, *Science*, 269, 198 (1995).
2. C.C Bradly, C.A. Sackett, J.J. Tollert and G. Hulet, *Phys. Rev. Lett. (PRL)* 75, 1687 (1995).
3. K.B. Davis, M.U. Mews, M.R. Andrews, N.J. Van Druten, D.S. Durfee, D.M. Kurn, and W. Ketterie, *Phys. Rev. Lett (PRL)* 75, 3969 (1995).
4. K. Huang, Statistical Physics (Wiley NewYork 1987) Chap. 10 (2^{nd} ed.).
5. Q. A. Wang, M. Pezeril and A.Le Mehanut, *Physica* A 278, 337(2000).
6. C.Tsallis, *J.Stat. Phys*. 52, 479 (1988).
7. E.MF Curado and C.Tsallis, *J.Phys. A, Math. Gen*. 24, L69 (1991).
8. C. Tsallis, *Physica* A 221, 227 (1995).
9. C. Tsallis, R.S. Mendes and A.R. Plastino, *Physica* A261, 534 (1998).
10. F. Buyukaili and D. Demirhan, *Phys. Lett*. A181, 24 (1993).
11. Q.A Wang and A. Le. Mehante, *Phys. Lett*. A 242, 301 (1998).
12. D.S. Petrov and G.V. Shlyapnikov, *Phys. Rev*. A64, 012706 (2001).
13. P.A. Lu, N. Nagaosa and X.G. Wen, *Rev. Mod. Phys*. 78, 17 (2006).
14. I.B. Spielman, W.D. Phillips and J.V. Porto, *Phys. Rev. Lett. (PRL)* 98, 080404 (2007).
15. M. Oashnii, H. Perrin and V. Lorent, *Phys. Rev. Lett. (PRL)* 105, 095302 (2010).